Sampling the transient AEM decay

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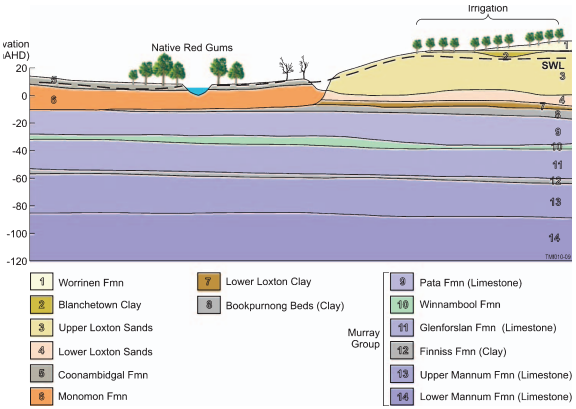
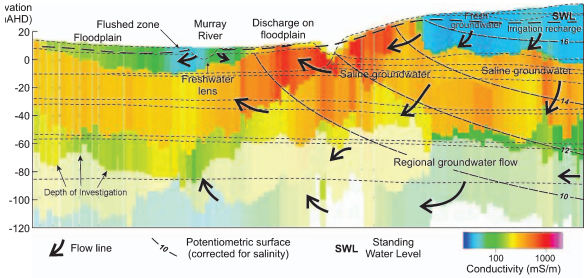
Motivation

Airborne electromagnetic surveys are established methods of mapping variation of electrical conductivity in the subsurface. These variations can be interpreted to derive hydrological information.





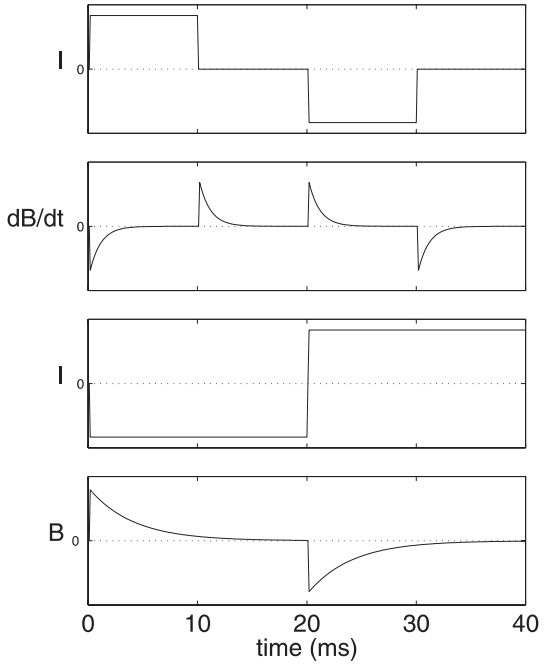
Some of the features we attempt to resolve from AEM data are very subtle. Research in this talk is aimed at better resolution of the earth through alternate sampling of the transient decays and thereby improve the quality of interpretation of AEM data.

Data collection

We consider *Tempest*-like systems where data are collected as a time series. The main processing steps are:-

1. reduction – attenuation of sferic, powerline, VLF & coil motion noise;
2. deconvolution of system response;
3. removal of primary field; and
4. conversion to square-wave **B**-field response

It is at step 4 that we have the opportunity to determine how we are going to sample the transient decay.



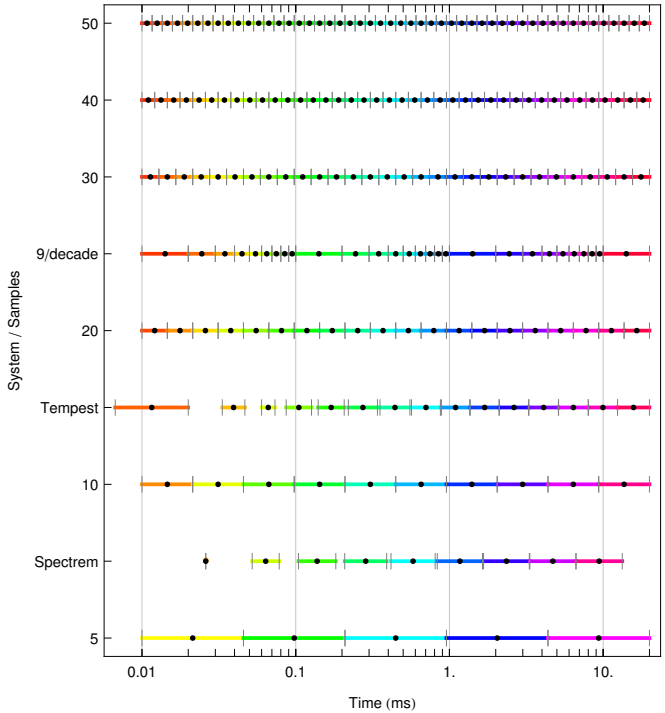
Actual transmitter current waveform

Measured ground response (dB/dt)

Transmitter current waveform for processed data

Ground response (B) in processed data

Sampling schemes



Window parameters are a compromise between accurately-sampling a wide range of earth responses (requires more windows) and minimising noise (requires long windows).

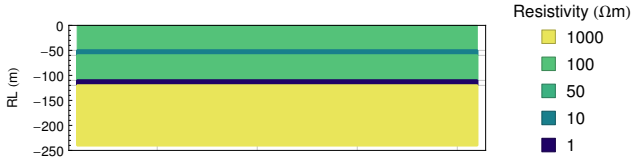
Black dots denote windows centres, vertical lines denote the start and end of each window, and colours are used to highlight different windows.

Of the nine schemes of sampling the transient decay that we consider, two model commercial systems (*Tempest* & *SPECTREM*$_{2000}$) while one employs linearly-positioned windows. The remainder position gates logarithmically in time.

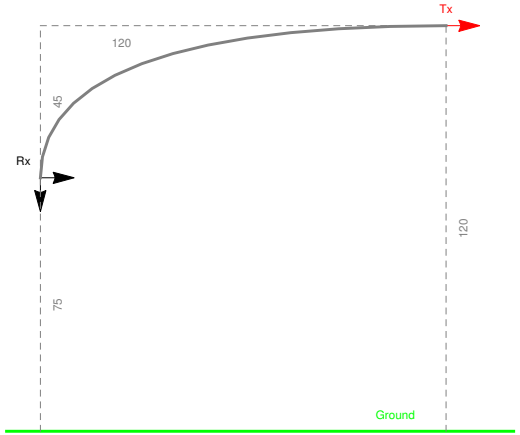
Systems are used for illustrative purposes only. We are not interested in benchmarking.

An example . . .

Consider a 1D model consisting of two 10 m thick conductors separated by 50 m in a moderately-resistive regolith over a resistive basement. The deeper conductor is $\times 10$ more conductive than the shallower one We consider AEM systems with a *Tempest*-like system geometry which employ a 25 Hz waveform and consider the square-wave **B**-field response.

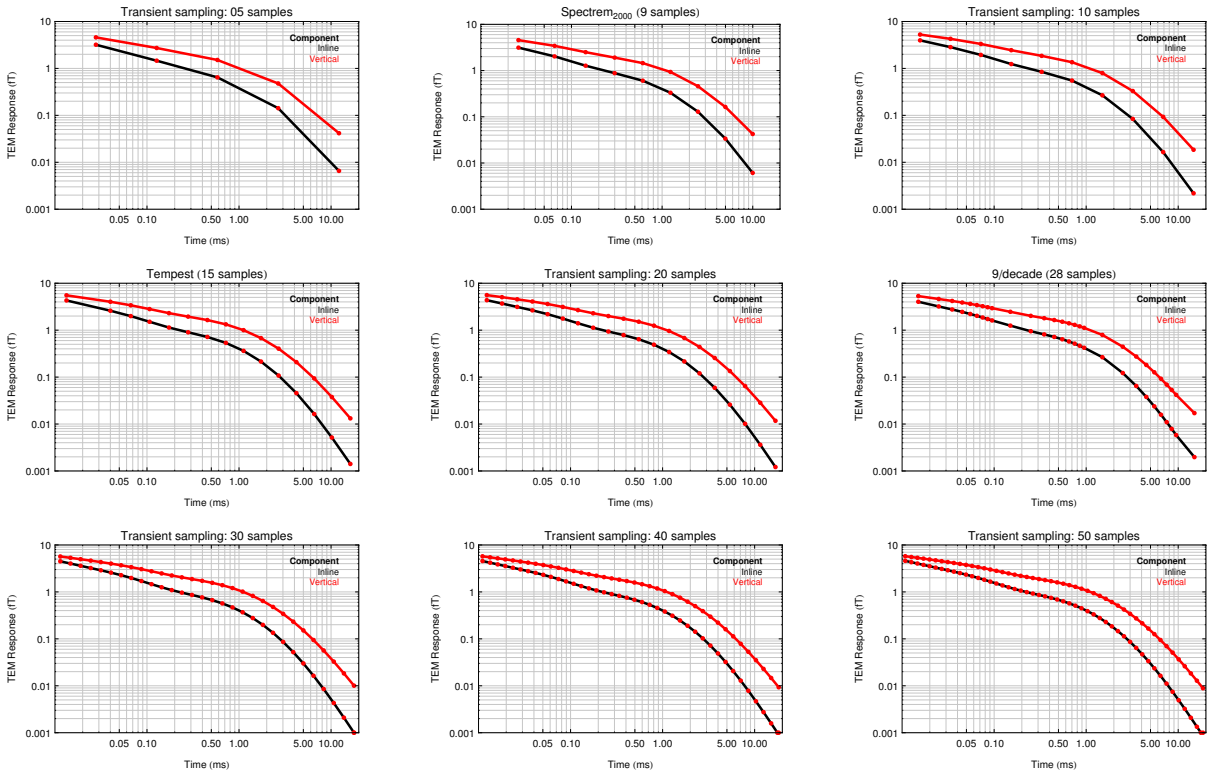


(a) Example model



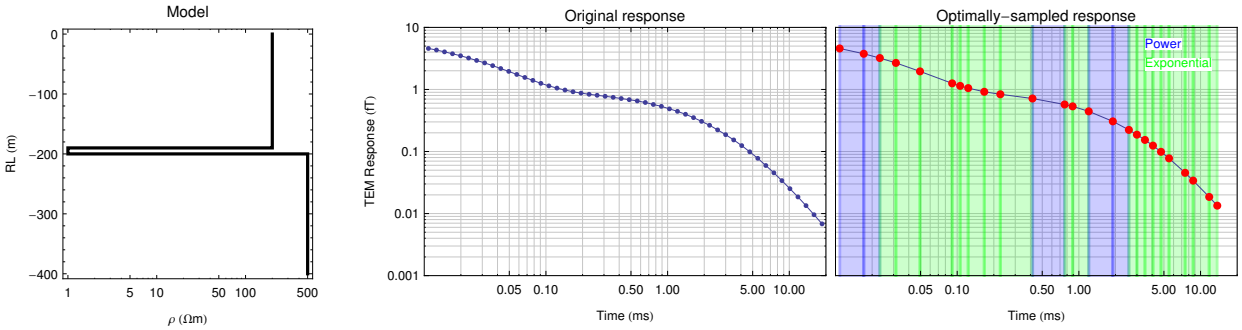
(b) Prospecting system

Model response using different sampling schemes . . .



Optimal sampling

We refer to an 'optimal' sampling scheme as one which samples a transient decay so that portions of the decay with differing $\frac{\partial B}{\partial t}$ are captured with an appropriate number of samples. In this work, we use a combination of exponential and power-law fitting. Clearly, there are alternative ways of summarising the decay.



The figure compares original and 'optimally-sampled' responses (with 1% noise added) for the vertical component response of the model. In comparison to the original 50-gate response, the 'optimally-sampled' response has 25 samples. Optimal sampling has characterised the decay in terms of five exponential decays and 19 power-law decays. Reducing the fitting tolerance (from $R^2 = 0.99999$) would result in the optimally-sampled response having fewer gates.

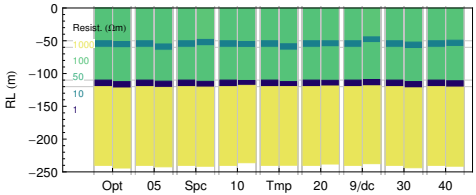
Methodology

A natural way to examine effects of employing different sampling schemes, is to compare results of inverting data for a layered earth, since this is how data are most commonly interpreted. The procedure below was used:-

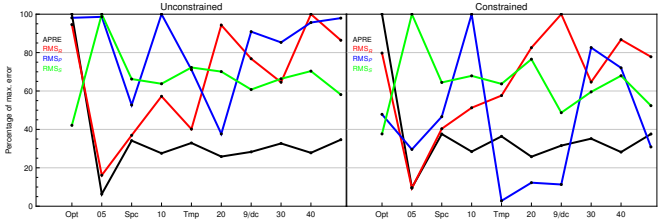
1. Calculate the model's forward response;
2. Add noise: 1, 2, 5 and 10% of signal amplitude in particular gate;
3. Invert data: we refine the model twice from an initial model 'close' to the correct model; and
4. Analyse results
   - Average Predicted Residual Error (APRE)
   - $\mathrm{RMS}_{\mathrm{Response}}$
   - $\mathrm{RMS}_{\mathrm{Parameters}}$
   - $\mathrm{RMS}_{\mathrm{Conductance}}$

We consider unconstrained inversions, and inversions in which layer thicknesses were constrained to their correct values. There are many effects that can be illustrated; we concentrate on a few . . .

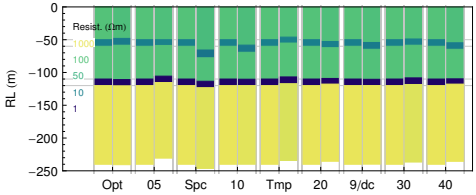
Inversion results (5% noise)



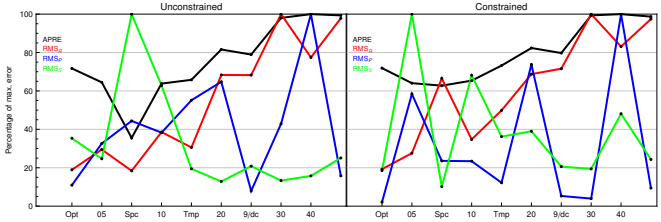
(a) Inline (unconstrained)



(b) Inline error metrics



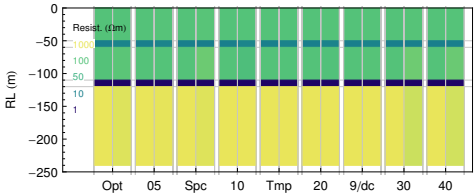
(c) Vertical (unconstrained)



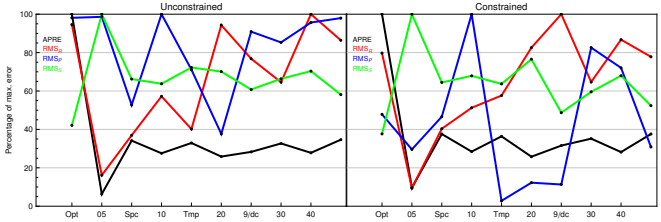
(d) Vertical error metrics

Different sampling schemes influence the inversion results . . .

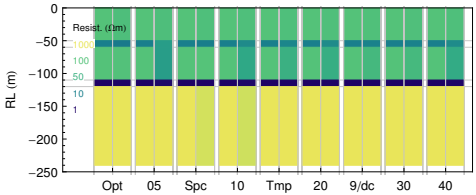
Inversion results (5% noise)



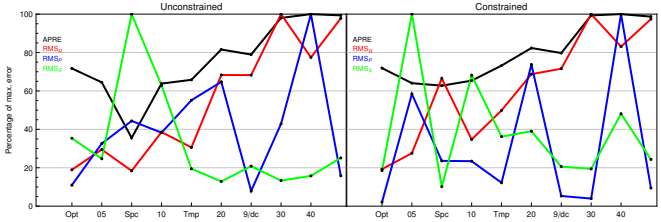
(a) Inline (k constrained)



(b) Inline error metrics



(c) Vertical (k constrained)



(d) Vertical error metrics

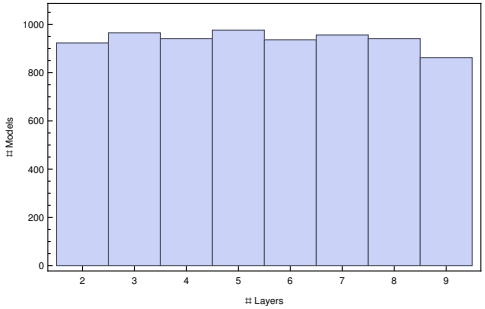
Different sampling schemes influence the inversion results . . .

More generally . . .

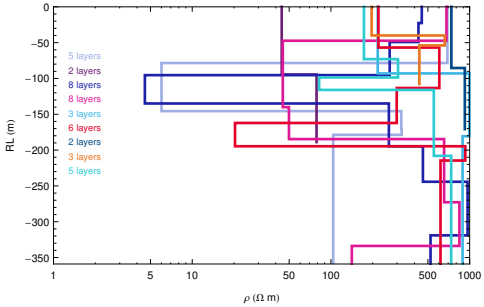
To test if an optimal sampling scheme is worthwhile, we perform the same experiment on a distribution of randomly-generated models. We consider an *n*-layered earth with $2 \leq n \leq 9$. We vary layer resistivity, ρ, and thickness k:-

$$0.1 \leq \rho \leq 1000\ \Omega\text{m}$$
$$1.0 \leq k \leq 100\ \text{m}$$

We compute the forward response of 7500 models taken from a uniform distribution, add noise, and invert for the appropriate layered earth. We do not enforce particular geologies.



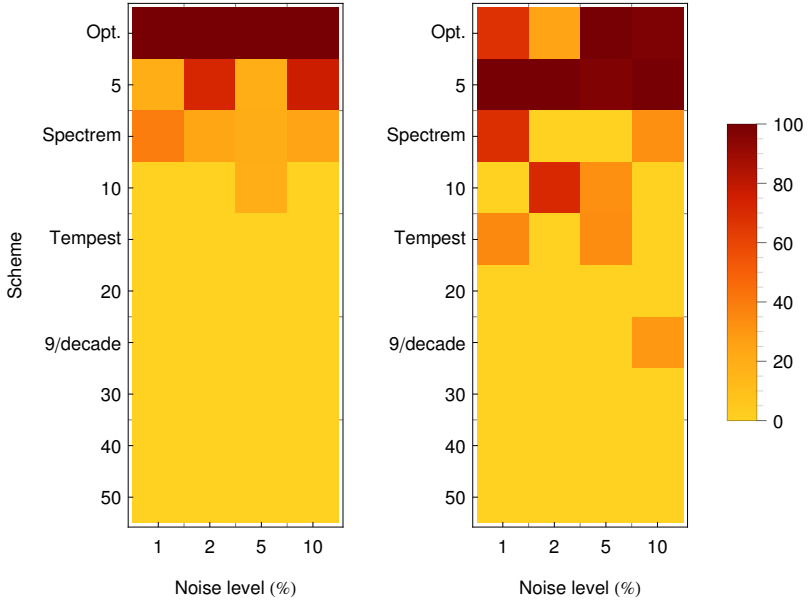
(a) Distribution of models



(b) Nine models drawn at random

Inversion results #1

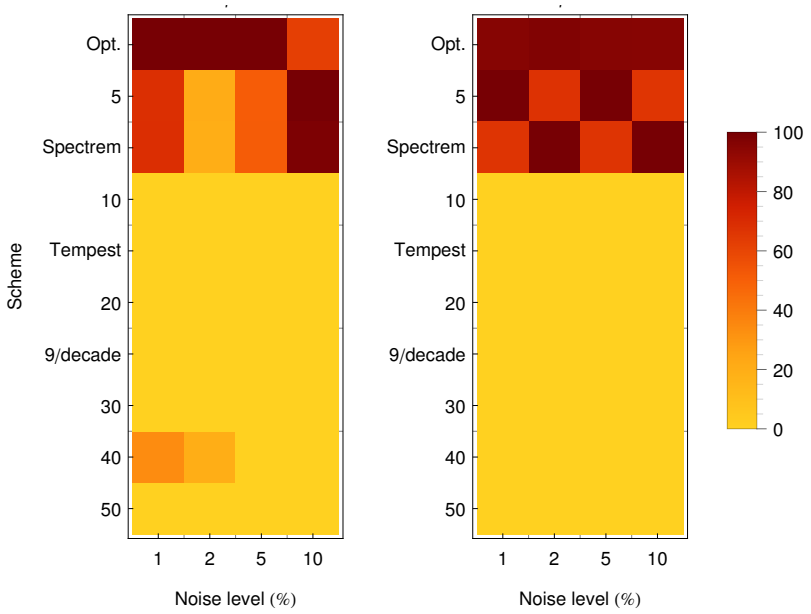
We present results as histograms which describe the error as a function of sampling scheme. Images below are normalised by maximum count – darker reds are better than yellows.



(a) APRE (Inline) **(b)** APRE (Vertical)

Inversion results #2

A naive optimal sampling scheme *generally* results in inverted models with lower APRE *and* more accurate models (lower RMS_P).



(a) RMS_P (Inline) **(b)** RMS_P (Vertical)

Conclusions

1. How the transient decay is sampled *does* have an effect on what we can interpret.
2. The sampling scheme has more of an effect in high-noise environments than in low-noise environments.
   - In high noise environments, lower APRE errors are seen when few windows are used
   - In high noise environments, more accurate (lower RMS_P) models can be inverted when more windows are used
3. There may be merit in processing time-series data so that it is fit-for-purpose, so that it accurately captures the decay of magnetic fields in the earth in as few gates as possible *and* provides a noise estimate.

Outlook

- How best to summarise a transient decay?
  - Sum of exponential decays?
  - Combination of exponential & power-law decays?
- End goal is to characterise noise on a fiducial-by-fiducial basis
  - Quiet data not degraded by noisy data
  - Noisy data not improperly characterised by quiet data

Thank You

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Codes . . .

Forward modelling used *LeroiAir* (in layered-earth mode) while inverse modelling used *Airbeo*. Both codes are part of a suite of electromagnetic modelling codes that were developed by CSIRO during nearly 30 years of industry-supported research. The suite includes programs to forward & inverse model 1, 2 and 3D EM responses. All codes were placed into the public domain in 2010.

Codes can be downloaded from AMIRA or SourceForge. They can be found by searching for 'P223 AMIRA'.

Parameter resolution matrices

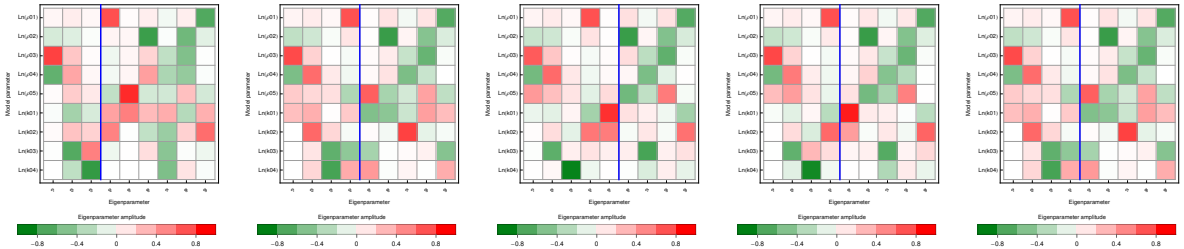
A useful concept is that of the resolution matrix. For a problem with M data points and a model with N parameters, the Jacobian

$$J = \frac{\partial d_i}{\partial p_j} \tag{1}$$

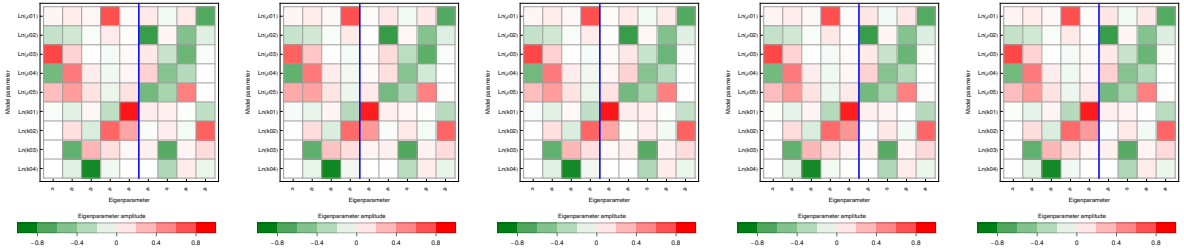
has a singular value decomposition USV^T. The $M \times M$ matrix U is the data resolution matrix while the $N \times N$ matrix V is known as the parameter resolution matrix. The matrix S contains the eigenvalues.

Some authors present $V^* = VV^T$. We plot V which enables us to determine how a particular parameter is resolved e.g. in isolation, or as a product or quotient.

Parameter resolution (1% noise)



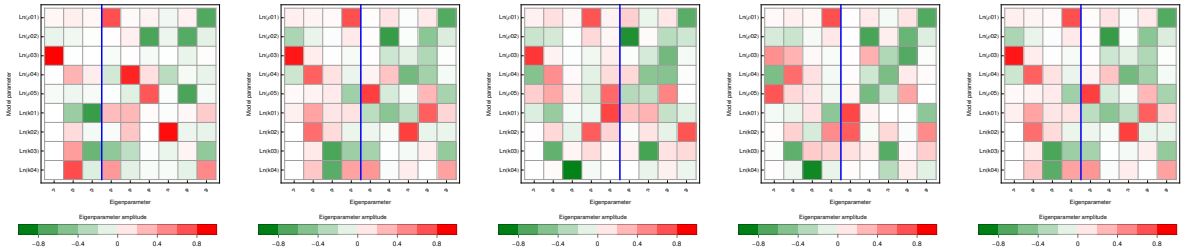
(a) Optimal **(b)** 5 gates **(c)** Spectrem **(d)** 10 gates **(e)** Tempest



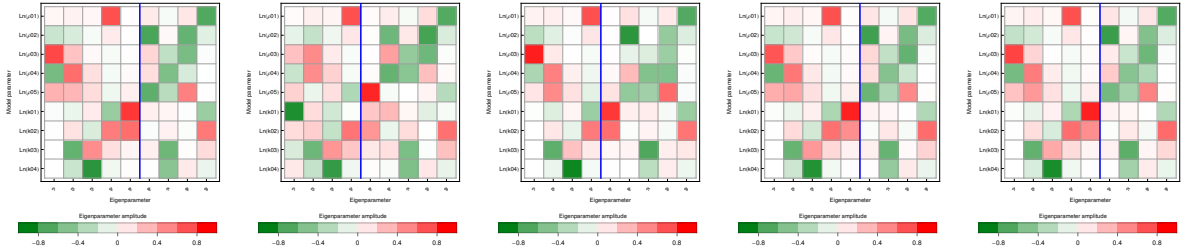
(f) 20 gates **(g)** 9/decade **(h)** 30 gates **(i)** 40 gates **(j)** 50 gates

Blue line indicates 0.8 damping factor; eigenparameters to the right of this line are not well resolved

Parameter resolution (10% noise)



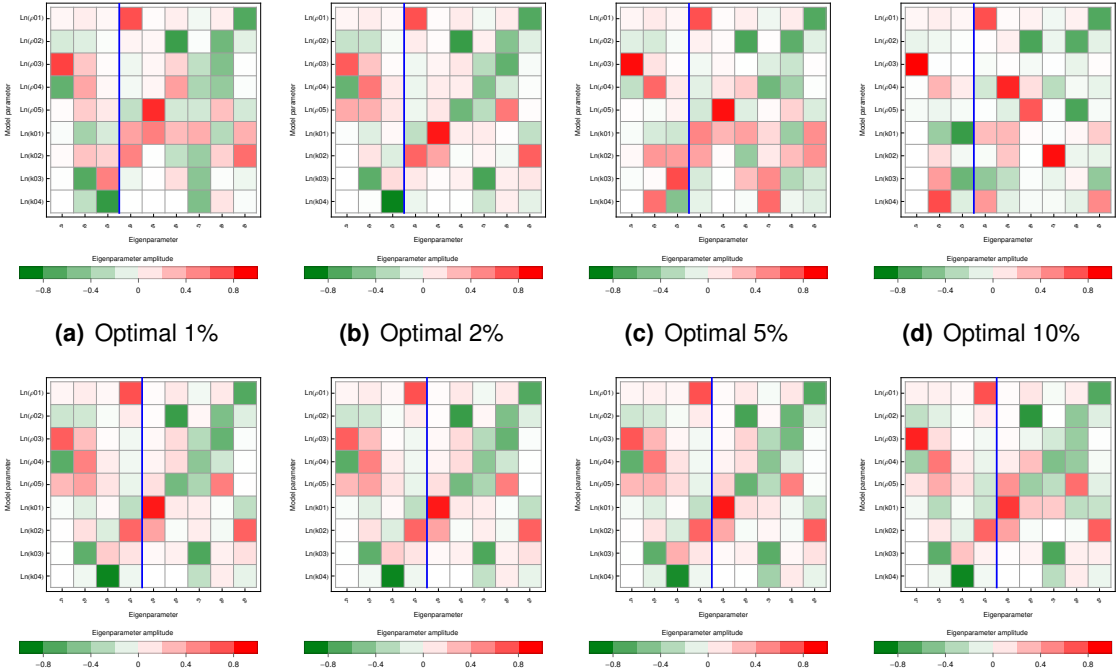
(a) Optimal **(b)** 5 gates **(c)** Spectrem **(d)** 10 gates **(e)** Tempest



(f) 20 gates **(g)** 9/decade **(h)** 30 gates **(i)** 40 gates **(j)** 50 gates

Blue line indicates 0.8 damping factor; eigenparameters to the right of this line are not well resolved

Parameter resolution (vertical component)



(a) Optimal 1% **(b)** Optimal 2% **(c)** Optimal 5% **(d)** Optimal 10%

(e) Tempest 1% **(f)** Tempest 4% **(g)** Tempest 5% **(h)** Tempest 10%

Average predicted residual error (APRE)

Airbeo's standard error metric is average predicted residual error which is defined

$$APRE = \sqrt{\frac{1}{M}\sum_{i=1}^{M} H_i (D_i - Y_i)^2} \tag{2}$$

$$H_i = \left\{ 1 - \sum_{j=1}^{N} U_{i,j}^2 \right\}^{-1} \tag{3}$$

where D_i and Y_i are the i-th data and model response respectively. The matrix U comes from the SVD of the Jacobian, J which expresses the change in model response d with change in model parameter p

$$J = \frac{\partial d_i}{\partial p_j} \tag{4}$$

$$= USV^T \tag{5}$$

where U is a $M \times M$ orthogonal matrix of data space eigenvectors, S is a diagonal $M \times N$ matrix containing singular values of J, V is an $N \times N$ matrix of model space eigenvectors and T indicates matrix transpose. There are M data points and N model parameters.

Thank You

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